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THEORY OF TRANSPORT OF ROCK PARTICLES DURING DRILLING WITH CONSIDERATION OF WATER ABSORPTION AND INFLOW

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An approximate one-dimensional theory of the process of transport of heavy solid rock particles by the flow of drilling mud in a vertical well is proposed.

The process of transport of solid rock particles by drilling mud plays an important role in the technological cycle of drilling. Imperfect bottom-hole flushing leads to collapse and shutdown of drilling. It is necessary to know the distribution of the concentration of solid particles in the annular space of the well to select the most efficient trouble-free drilling practices. The solid particles of fractured rock carried to the surface by the drilling mud have an order of 10^{-3}-10^{-7} m. Drilling mud is an aqueous suspension of clay with various additives; its viscosity is of the order of 20-200 cP. The free-fall velocity of the heavy rock particles in the mud does not exceed 1 m/sec in order of magnitude, and the characteristic drilling-mud velocity has an order of 10 m/sec. Therefore we will consider that the velocity of the solid particles is equal to the mud velocity. In addition, we will assume the well is vertical and the process is one-dimensional.

On the basis of the adopted assumptions we obtain the following equations of mass transport:

$$S\left(\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}\right)=-cQ^{-}(x,\ t)+g(x,\ t), \tag{1}$$

$$vS=Q_0(t)+\int\limits_0^x Q^{+}(x,\ t)\,dx-\int\limits_0^x Q^{-}(x,\ t)\,dx. \tag{2}$$

Usually Q^+ and Q^- are observed on the exposed, uncased section of the well and are associated with various complications occurring during drilling (for example, as a consequence of fractures, creep, or low strength of the rocks and lost circulation, water inflow, cave-in, etc., caused by these factors). Henceforth Q^-, Q^+, Q_0, and g are considered known.

Let us examine some solutions of Eq. (1).

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1. Cased Section of Well. When $x \geqslant x_0$, $t > t_0$ let there occur the following conditions of ideal trouble-free operation with an annular cross section of constant area:

$$Q^+ = Q^- = 0, \; g = 0, \; v = \text{const}. \tag{3}$$

The general solution of Eq. (1) in this case has the form

$$c = F(\xi), \; \xi = x - \int v(t)\,dt, \; vS = Q_0(t), \tag{4}$$

where $F(\xi)$ is an arbitrary function.

When $x = x_0$ let $c = c_0(t)$ (Cauchy problem). Hence from (4) we obtain the following condition for determining $F(\xi)$:

$$F(\xi) = c_0(t), \; \xi_0 = x_0 - \int v(t)\,dt. \tag{5}$$

We denote $f = \int v(t)dt$ and the function $t = f^*(f)$ inverse to it. From (5) we find

$$t = f^*(x_0 - \xi_0), \; f = x_0 - \xi_0. \tag{6}$$

Substituting into (5), we obtain

$$F(\xi) = c_0[f^*(x_0 - \xi)], \tag{7}$$

and the solution of the Cauchy boundary-value problem is written so:

$$c = c_0[f^*(x_0 - \xi)], \; \xi = x - \int v(t)\,dt. \tag{8}$$

For example, when v = const at times t ($t_1 > t > t_0$) we have

$$c_0 = c_0\left(t - \frac{x - x_0}{v}\right) \text{ for } t_1 > t - \frac{x - x_0}{v} > t_0. \tag{9}$$

This is a steady operating regime of the pump during time $t_1 > t > t_0$ (and "steady" complications in section $0 < x < x_0$ at times $t_1 > t > t_0$).

When $x_0 = 0$ the solution (9) corresponds to the same pump regime in ideal trouble-free well operation (under varying, unsteady bottom-hole operating conditions, for example, as a consequence of the geological conditions).

If, furthermore, the condition $c = c_0$ = const for $t_1 > t > t_0$ is fulfilled, then it follows from (9) that

$$\text{for } t_1 > t - \frac{x - x_0}{v} > t_0 \quad c = c_0 = \text{const}. \tag{10}$$

This simple solution corresponds also to steady entry of suspended particles into cross section $x = x_0$. When $x_0 = 0$ it corresponds to an ideal trouble-free steady operating regime of the well and pump under lowland geological conditions.

We will now study the case when v, g, and Q^- are arbitrary functions of t which are independent of x. We will seek the general solution of Eq. (1) in the form

$$c = F(\xi, t), \; \xi = x - \int v(t)\,dt, \tag{11}$$

where F is an arbitrary function. We find

$$S\frac{\partial F}{\partial t} = -FQ^- + g. \tag{12}$$

Integrating, we obtain

$$c = \exp\left[-\frac{1}{S}\int Q^-(t)\,dt\right]\left\{\frac{1}{S}\int g(t)\exp\left[\frac{1}{S}\int Q^-(\tau)\,d\tau\right]dt + C(\xi)\right\}. \tag{13}$$

Here $C(\xi)$ is an arbitrary function determined from the boundary condition: when $x = 0$, $c = c_0(t)$.

Analogously to the preceding, we find

$$C(\xi) = G[f^*(-\xi)]. \tag{14}$$

Here $f = \int v(t)dt$; $t = f^*(f)$ $(f^*(f(t)) = t)$;

$$G(\alpha) = c_0(\alpha)\exp\left[\frac{1}{S}\int Q^-(\alpha)\,d\alpha\right] - \frac{1}{S}\int g(\alpha)\exp\left[\frac{1}{S}\int Q^-(\alpha)\,d\alpha\right]d\alpha.$$

2. Lost Circulation. Let lost circulation occur only at point $x = x_*$ of the uncased section of the well and water inflow and similar phenomena be absent:

$$g=0,\ Q^-(x,\ t)=Q^-(t)\,\delta(x-x_*),$$
$$Q^+(x,\ t)=0,\ v=\begin{cases} v_0 \text{ for } 0<x<x_*, \\ v_0-\dfrac{1}{S}Q^-(t) \text{ for } x_*<x<x_0. \end{cases} \tag{15}$$

Here $\delta(x - x_*)$ is the Dirac delta function; $Q^-(t)$ is the mud flow rate from the well in unit time. The flow of the drilling mud to the bottom of the hole is considered steady, i.e., $v_0 = \text{const}$.

Equation (1) will take the form

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=-c\,\frac{1}{S}Q^-(t)\,\delta(x-x_*) \tag{16}$$

or in another, detailed notation:

$$\frac{\partial c}{\partial t}+v_0\frac{\partial c}{\partial x}=0 \text{ for } 0<x<x_*,$$
$$\frac{\partial c}{\partial t}+\left[v_0-\frac{1}{S}Q^-(t)\right]\frac{\partial c}{\partial x}=0 \text{ for } x_*<x<x_0, \tag{17}$$
$$c(x_*+0,\ t)-c(x_*-0,\ t)=0.$$

The last relationship expresses the physical condition of continuity of the concentration of suspended particles at singular point $x = x_0$.

The general solution of Eq. (16) or of Eqs. (17) equivalent to it, according to (4), has the following form:

$$c=\begin{cases} c_0\left(t-\dfrac{x}{v_0}\right) \text{ for } 0<x<x_*, \\ F(\xi) \text{ for } x_*<x<x_0, \end{cases} \tag{18}$$
$$\xi=x-v_0t+\frac{1}{S}\int Q^-(t)\,dt.$$

Here the argument of the arbitrary function $c_0(\alpha)$ is selected so that the boundary condition at the bottom of the hole is satisfied: when $x = 0$, $c = c_0(t)$. Function $F(\xi)$ is found from the condition of joining of the solutions at point $x = x_*$ (the last equation of (17)):

$$c_0\left(t-\frac{x_*}{v_0}\right)=F\left(x_*-v_0t+\frac{1}{S}\int Q^-(t)\,dt\right).$$

Hence, having denoted $f = (1/S)\int Q^-(t)dt$, $t = f^*(f)$ so that $f^*(f(t)) = t$, we obtain

$$F(\xi)=c_0\left(-\frac{\xi}{v_0}+\frac{1}{v_0S}\int Q^-(t)\,dt\right), \tag{19}$$

consequently, according to (18), we have

$$\text{for } 0<x<x_0 \quad c=c_0\left(t-\frac{x}{v_0}\right), \tag{20}$$

as in the absence of lost circulation. However, the total flow of the mass of suspended particles, obviously, will decrease by the quantity $c/Q^-(t)$ when $x > x_*$.

This result, obtained from physical considerations, is easily generalized to the case of arbitrary lost returns, i.e., arbitrary function $Q^-(x, t)$, in the following way: the one-dimensional distribution of the concentration c in time does not depend on arbitrary nonselective lost returns. Therefore, without loss of generality the one-dimensional equation of mass transform (1) can be written so:

$$\frac{\partial c}{\partial t}+v\frac{\partial c}{\partial x}=\frac{1}{S}g(x,\ t). \tag{21}$$

In the expression of v as a function of x and t in this case also, naturally, lost circulation is not taken into account.

3. Transport of Particles with Consideration of Water Inflow. Let water inflow occur only at point $x = x_*$ of the uncased section of the well:

$$g(x, t) = g(t)\,\delta(x - x_*),$$
$$v = \begin{cases} v_0(t) & \text{for } 0 < x < x_*, \\ v_0(t) + \dfrac{1}{S} g(t) & \text{for } x_* < x < x_0. \end{cases} \tag{22}$$

Here g(t) is the flow of extraneous fluid into the well in unit time; $v_0(t)$ is a given function corresponding to arbitrary, generally speaking, unsteady flow of the fluid to the bottom of the hole.

Thus under conditions (22) the equation of mass transport (21) has the form

$$\frac{\partial c}{\partial t} + v(x, t)\frac{\partial c}{\partial x} = 0 \quad (0 < x < x_0) \tag{23}$$

with the condition of conservation (condition on a strong discontinuity)

$$c(x_* - 0, t) = \left[1 + \frac{g(t)}{v_0(t) S}\right] c(x_* + 0, t) \tag{24}$$

at singular point $x = x_*$ (point of discontinuity of the solution).

The general solution of (23) has the form:

when $0 < x < x_*$

$$c = c_0[t(-\xi)], \quad \xi = x - \int v_0(t)\,dt, \tag{25}$$

where

$$\int v_0(t)\,dt = \alpha, \quad t = t(\alpha) \quad (t(\alpha(t)) = t),$$

when $x_* < x < x_0$

$$c = F(\xi), \quad \xi = x - \int v_0(t)\,dt - \frac{1}{S}\int g(t)\,dt, \tag{26}$$

where

$$\frac{1}{S}\int g(t)\,dt = \beta, \quad t = t(\beta) \quad (t(\beta(t)) = t).$$

Here $c_0(t)$ is the concentration of suspended particles at the bottom of the hole, i.e., $c = c_0(t)$ when $x = 0$; $F(\xi)$ is a function which should be determined from the condition of conservation at the point of discontinuity $x = x_*$.

We find

$$c_0[t(-\xi_*)] = \left(1 + \frac{g}{v_0 S}\right) F(\xi_* - \beta), \tag{27}$$

where

$$\xi_* = x_* - \int v_0(t)\,dt = \xi_*(t).$$

Having denoted

$$A(t) = \frac{v_0(t) S}{g(t) + v_0(t) S} c_0\{t[-\xi_*(t)]\},$$
$$\tau = \xi_*(t) - \beta(t), \quad t = t(\tau) \quad (t(\tau(t)) = t),$$

on the basis of (27) we obtain

$$F(\xi) = A(t(\xi)). \tag{28}$$

Equations (25), (26), and (28) give the exact solution of the stated problem.

Let water inflow occur uniformly on well section $0 < x < x_0$:

$$g(x, t) = g(t) \text{ for } 0 < x < x_0,$$
$$v = v_0(t) + xS^{-1}g(t) \text{ for } 0 < x < x_0. \quad (29)$$

Here g(t) is the inflow of extraneous fluid into the well per unit length of it.

In case (29) the mass-transport equation (21) has the form

$$\frac{\partial c}{\partial t} + \left[v_0(t) + \frac{1}{S} xg(t)\right] \frac{\partial c}{\partial x} = \frac{1}{S} g(t) \; (0 < x < x_0). \quad (30)$$

We divide both sides of this equation by g(t) and introduce the new notations:

$$\tau = \frac{1}{S} \int g(t)\,dt, \quad v_1(\tau) = S \frac{v_0[t(\tau)]}{g[t(\tau)]}, \; c = \tau + c_1(x, \tau). \quad (31)$$

Hence from (30) we obtain

$$\frac{\partial c_1}{\partial \tau} + [x + v_1(\tau)] \frac{\partial c_1}{\partial x} = 0. \quad (32)$$

The characteristic equations related to Eq. (32) will be the following:

$$\frac{dc_1}{ds} = q + p(x + v_1(\tau)), \; \frac{d\tau}{ds} = 1, \; \frac{dp}{ds} = -p, \quad (33)$$
$$\frac{dq}{ds} = -pv_1'(\tau), \; \frac{dx}{ds} = x + v_1(t),$$

where s is an arbitrary parameter, $p = \partial c_1/\partial x$, $q = \partial c_1/\partial \tau$, so that Eq. (32) takes the form

$$q + p[x + v_1(\tau)] = 0. \quad (34)$$

The integrals of the first three equations of (33) are obvious:

$$c_1 = c_{10}, \; \tau = s + \tau_0, \; p = p_0 e^{-s}. \quad (35)$$

Here c_{10}, τ_0, p_0 are arbitrary constants. Substituting integrals (35) into the fourth and fifth equations of (33), we obtain with consideration of (34)

$$\frac{dq}{ds} = -p_0 e^{-s} v_1'(s + \tau_0), \; \frac{dx}{ds} = 2x + \frac{q}{p}. \quad (36)$$

The solution of these equations has the form

$$q = q_0 + p_0 \int e^{-s} v_1'(s + \tau_0)\,ds,$$
$$x = x_0 e^{2s} + e^{2s} \int Q(s) e^{-2s}\,ds. \quad (37)$$

Here

$$Q(s) = \frac{q_0}{p_0} e^s - e^s \int e^{-s} v_1'(s + \tau_0)\,ds;$$

q_0 and x_0 are arbitrary constants.

The family of curves on plane $x\tau$

$$x = e^{2\tau} \left(C - \int_{\tau_0}^{\tau} e^{-\tau} d\tau \int_{\tau_0}^{\tau} v_1'(\tau) e^{-\tau} d\tau\right), \quad (38)$$

where

$$C = e^{-2\tau_0} [2x_0 + v_1'(\tau_0)],$$

represents, according to the solution (35), (37) obtained, a certain family of characteristic curves (see Fig. 1). The relationships

$$c_1 = c_{10}, \; \frac{\partial c_1}{\partial x} = p_0 e^{\tau_0 - \tau}, \; \frac{\partial c_1}{\partial \tau} = q_0 - p_0 e^{\tau_0} \int_{\tau_0}^{\tau} e^{-\tau} v_1'(\tau)\,d\tau, \quad (39)$$

where

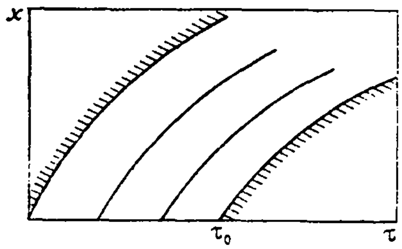

Fig. 1. Characteristic curves with parameters c_{10}, p_0, τ_0.

$$q_0 + p_0 [x_0 + v_1(\tau_0)] = 0,$$

are valid along each of these characteristics.

Constants c_{10}, q_0, p_0 vary from one characteristic to another. Relationships (39) permit finding the solution of the boundary-value problem (Cauchy problem)

$$\text{for} \quad x = 0 \quad c_1 = c_{10}(\tau). \tag{40}$$

As is seen, c_1 will be constant along each characteristic, the value of the constant being determined by condition (40) at the point of axis τ formed by the intersection of axis τ with the corresponding characteristic.

We will examine a particular case of this problem:

$$g(t) = g = \text{const}, \ v_0(t) = v_0 = \text{const} \ (0 < t < t_0). \tag{41}$$

This is a steady regime of delivery of the drilling mud to the bottom and water inflow in the well in the time interval from zero to t_0. In this case, according to (31), the family of characteristics (38) will be the following:

$$x = Ce^{2\tau}. \tag{42}$$

The relationships

$$c_1 = c_{10}, \ \frac{\partial c_1}{\partial x} = p_0 e^{\tau_0 - \tau}, \ \frac{\partial c_1}{\partial \tau} = q_0,$$

where

$$q_0 + p_0\left(x_0 + \frac{v_0}{g} S\right) = 0, \ x_0 + v_1(\tau_0) = 0.$$

are valid along each characteristic on the basis of (39). The general solution of mass transport has the form

$$c = \frac{1}{S} gt + F(\xi), \quad \xi = \frac{S}{g} \ln\left(v_0 + \frac{g}{S} x\right) - t, \tag{43}$$

where $F(\xi)$ is an arbitrary function.

The general solution of transport equation (21) is constructed analogously in the case when v is an arbitrary function of x but does not depend on t. In this case

$$c = F(\xi) + \ln v(x), \ \xi = \int \frac{dx}{v(x)} - t, \tag{44}$$

since

$$g(x, t) = S \frac{\partial v}{\partial x}.$$

In the general case the equation of transport of particles (21) is written in the most compact form:

$$\frac{\partial c}{\partial t} + v \frac{\partial c}{\partial x} = \frac{\partial v}{\partial x}$$
$$\left(v(x, t) = v_0(t) + \frac{1}{S} \int g(x, t)\, dx \right). \tag{45}$$

The characteristic equations related to Eq. (45) are the following:

$$\frac{dx}{dt} = v(x, t), \quad \frac{dc}{dt} = \frac{\partial v}{\partial x},$$
$$\frac{dp}{dt} = -p\frac{\partial v}{\partial x} + \frac{\partial^2 v}{\partial x^2}, \quad \frac{dq}{dt} = -p\frac{\partial v}{\partial t} - \frac{\partial^2 v}{\partial x \partial t}. \tag{46}$$

Here

$$p = \frac{\partial c}{\partial x}, \quad q = \frac{\partial c}{\partial t} \quad \left(q + pv = \frac{\partial v}{\partial x}\right).$$

The solution of the first equation of (46) determines a one-parameter family of characteristic curves on plane xt; this solution is found by standard numerical methods. In the case when function v(x, t) can be represented as the product $v(x, t) = v_1(x)v_2(t)$ (and also in certain other cases) it is easy to find a closed analytic solution of the first equation of (46) in an implicit form.

The second equation of (46) is solved in the general case in the form

$$c = c_0 + \int \frac{\partial v}{\partial x}\, dt.$$

This relationship is fulfilled on each characteristic; it permits finding numerically the field of concentration of solid particles in the well at any time on the basis of a given concentration at the boundary when x = 0.

The region of existence of a unique solution is determined by the structure of the family of characteristic curves, i.e., by the first equation of (46).

The solutions obtained can be used in practice for obtaining information about water breakthrough into a well and about lost circulation on the basis of the concentration of rock particles in the fluid flow observed on the surface.

NOTATION

c, volume concentration of solid particles; S, cross-sectional area of annular space; v, fluid velocity; Q_0, volume flow rate of drilling mud; Q^+, volume inflow of groundwaters into well; Q^-, volume outflow of drilling mud into rock; t, time; x, axis of well; g, volume inflow of new solid particles into well; δ, Dirac delta function.

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